The Fully Efficient Skating Stroke. Part 1: The Start.
c. P. J. Baum, December 1999.

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Introduction

Recently I looked at a nonlinear stroke starting from the (assumed) applied force and worked toward the resulting motion. The solution could only be estimated because there was no model for the cornering force. However, it was clear that the cornering pull-force would increase the skating efficiency. Consequently, here I approach the solution exactly backwards from the previous attempt. That is, I find stroke motions which are fully efficient and work backwards toward the forces that would create them. The approach is reasonably successful and leads to a detailed picture of one simple, fully efficient stroke, and the forces that drove it.

The Approach

One key to this approach is finding what would constitute a fully efficient stroke. I started from the linear accelerator, e.g. a bicycle moving straight forward and pushed by a force also facing straight forward. This is a one-dimensional problem and is easily described. If the force is F and the mass is M then the bicycle's acceleration is a=F/m, its velocity is v=a*t and its energy is Elin=(Mv^2)/2. Since there is only motion forward and I am not considering drag forces this linear acceleration will be fully or 100% efficient.

The method used here is to look for a two-dimensional (nonlinear) skating stroke which will always have the same energy as the linearly accelerated object (bicycle). Now the skater will produce sideways energy (Es) as well as forward energy (Ef). So to match the linear accelerator I require that the stroke satisfy

$$Ef + Es = Elin,$$

or

$$Vf^2 + Vs^2 = v^2 = (at)^2 = (F/m)^2 * t^2.$$

One solution to this energy equation is

$$Vf=(F*t/M)*\sin[Pi*t/(2*T)]$$

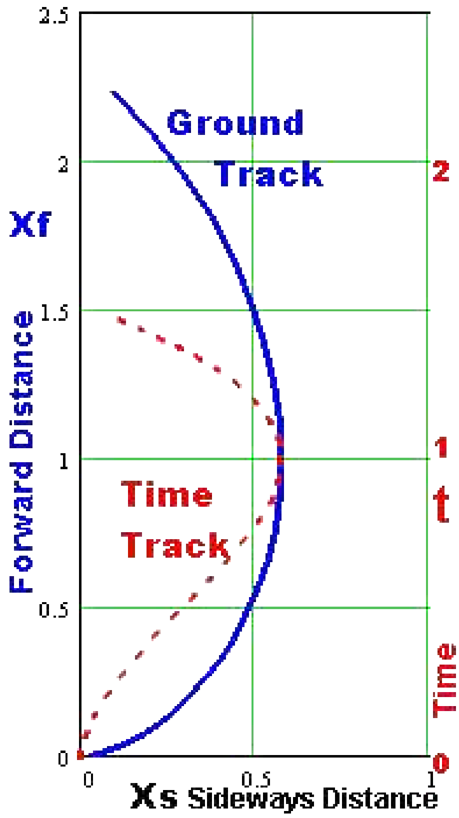
along with

$$Vs=(F*t/M)*\cos[Pi*t/(2*T)].$$

Here Pi =3.14..., t=time, T=time of maximum stroke width (T=1 in this model). For simplicity I have chosen a constant force, F, and express the results below in dimensionless form. This solution can be integrated for the distances travelled: Xs sideways, Xf forward. Every parameter can be expressed in terms of simple analytic functions for this stroke. Here I am using this solution only for the first stroke, however, as the stroke width grows larger with time and the solution becomes unrealistic for later times. There are other solutions but this one is quite suitable for

understanding the basic stroke mechanism.

Results

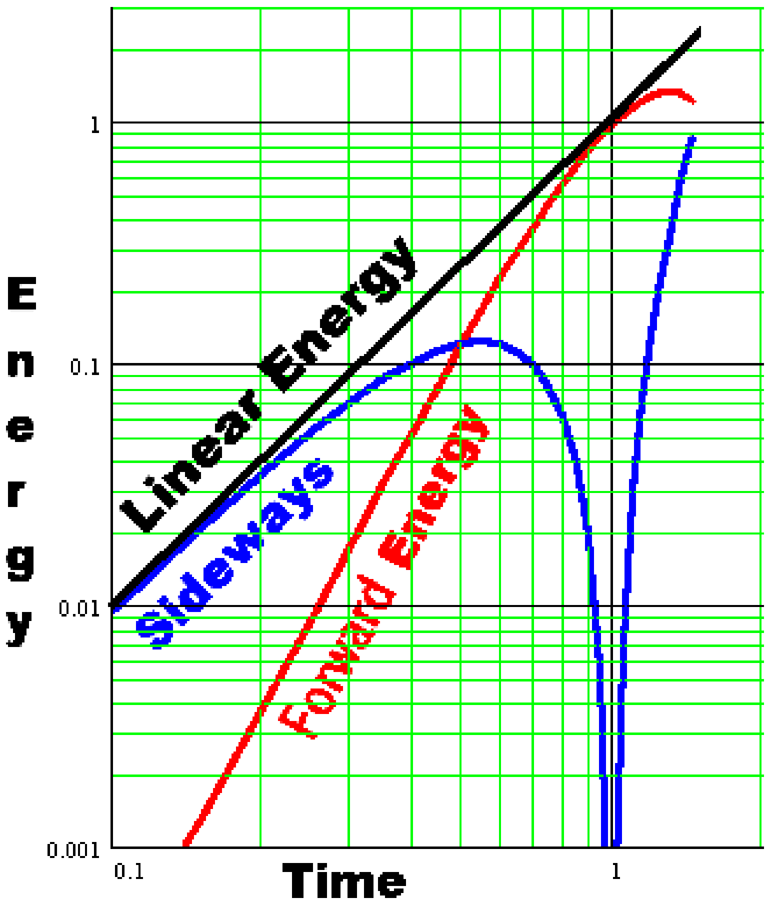


For this figure the velocity equations have been integrated to give the distances travelled. The Ground Track plots forward distance vs. sideways distance. The skate moves from the origin out to a maximum stroke width (Xs=1 at t=T=1) and it faces straight ahead there. As the skater is accelerating, the path covers a forward distance which is increasing as it returns toward the center line. The stroke is serpentine in nature and is similar to that used in the double-push.

The Time Track shows the sideways skate position as a function of time. This is what one would see video-taping the skater from in front or behind at a constant frames-per-second rate. Because of the acceleration the skater covers the second half of the stroke in a shorter time than the first half.

Here The velocity equations are used (squared) to form the forward, sideways, and linear energy as a function of time. The results are plotted on a logarithmic (log-log) scale and the energy from linear acceleration (bicycle) is shown as the black straight line.

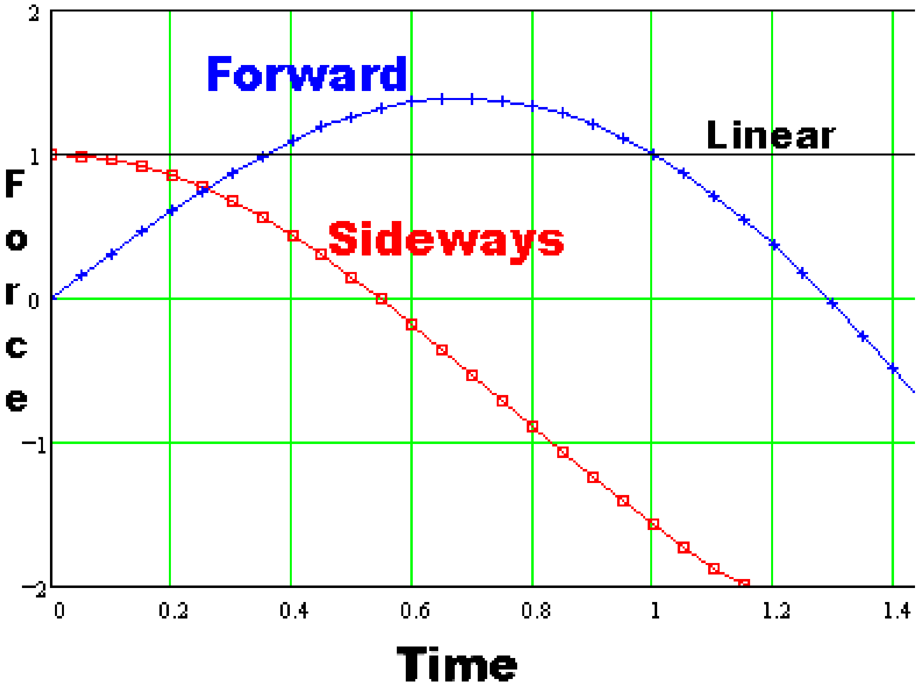
At the beginning of the stroke the energy is nearly all sideways but at the maximum stroke width (Xs=1 at t=1) the energy is all forward. And at this time *not only is the energy all forward but it is also exactly equal to the energy of the linear accelerator*. This energy transfer process from sideways to forward (the fully efficient single stroke) is the fundamental building block of speedskating.



Notice that the forward energy is continuously increasing up until t=1 after which it declines. This means that the forward velocity will also decrease after the maximum stroke width is reached at t=1. So the idea

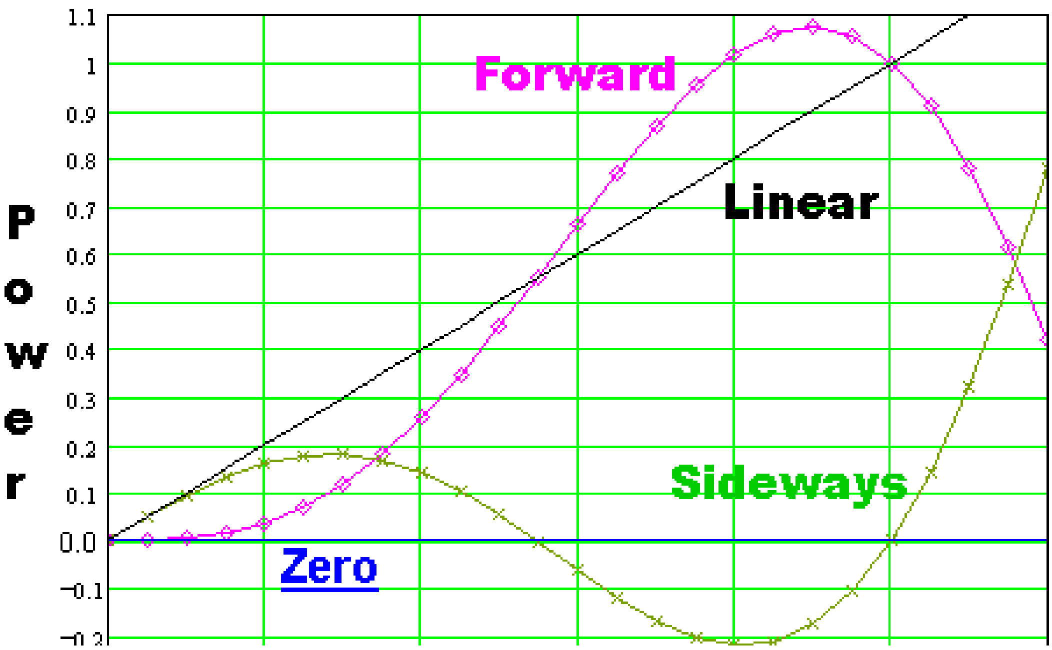
of "accumulation of speed" will need some refining. (The total speed is increasing continuously but its direction is oscillating and only points forward at the end of the stroke.)

Next we need to know what conditions produced this remarkable stroke. Differentiating the velocity equations yields the sideways and forward accelerations and forces (F=M/a). Plotted here, the sideways force starts out at the maximum (the constant linear force) while the forward force starts from zero. A little later the forward force exceeds the sideways force,

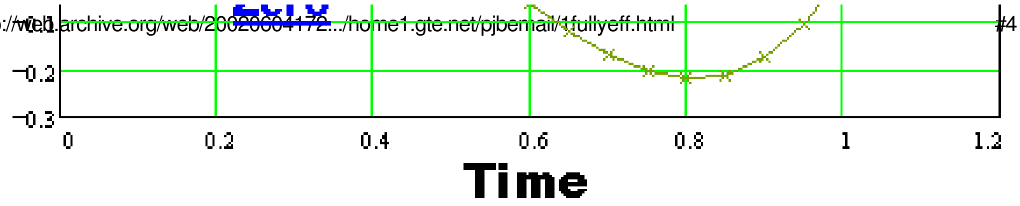


and at t=0.57 or just over halfway to the maximum stroke width the net sideways force changes sign. That is, the sideways force started out as a *push* force but changed to a *pull* force. And at the time T=1 (Xs=1) the forward force exactly equals the force of the linear accelerator.

Continuing on, the stroke Power can be calculated either from F*V or from time differentiation of the energy. The sum of the sideways and forward power add up to the power of the linear accelerator at all times. The power is first built up in the

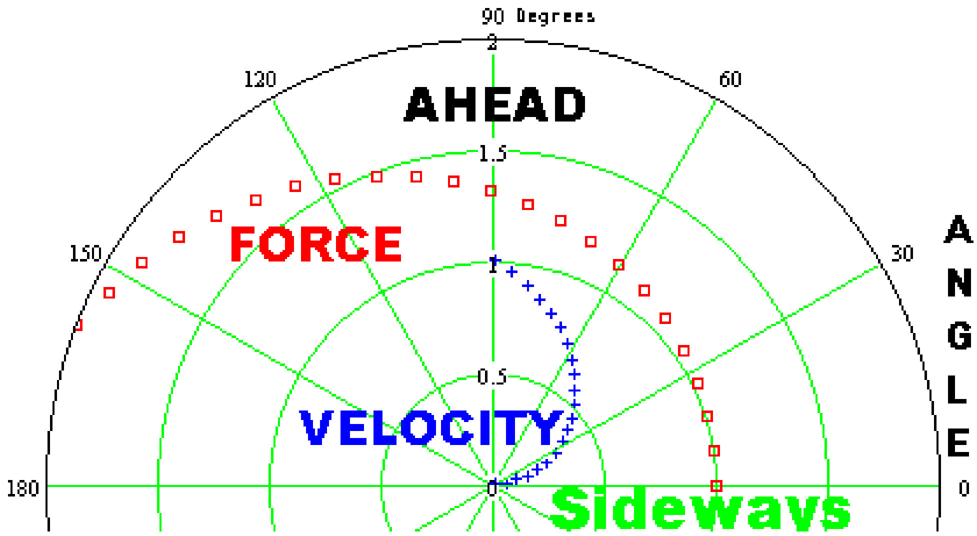


sideways motion and later in the forward motion.



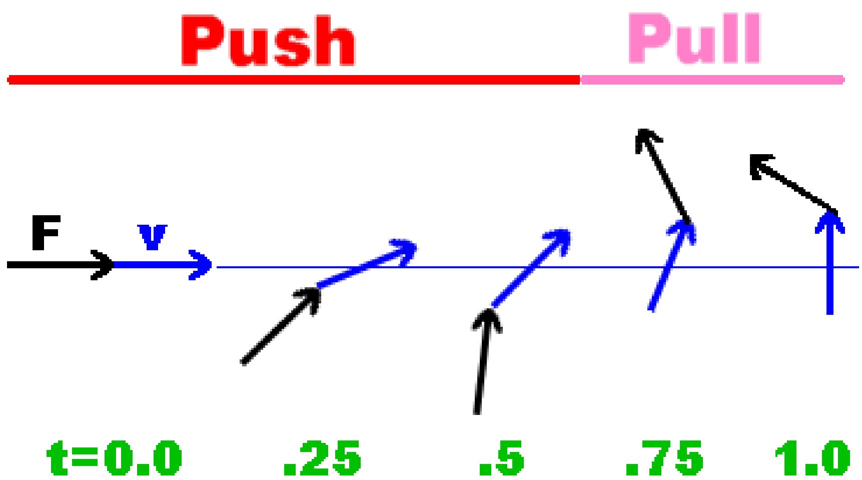
In fact the power is transferred from the sideways motion into the forward motion until the power is fully forward at t=T=1. In techno-jargon we could say that "the transverse (sideways) mode was energized and then power was pumped from the transverse mode into the axial (forward) mode".

Now we look at how the power is transferred between modes. From the vector forces and velocities the magnitude and direction (angle) of the total force and velocity vectors was calculated. As shown here the velocity (blue "+") starts out at the origin of the polar plot.



The force (red boxes) starts out sideways at an angle of 0 degrees. each "+" and "box" defines the end of the vectors (not drawn) which all start at the origin. Each adjacent "+" and "box" is separated in time units of 0.05. The skate faces straight ahead at t=1 (90 degrees) but by this time the force vector has rotated past 150 degrees. Consequently the force vector is rotating counterclockwise a little less than twice as fast as the velocity vector. It looks pretty clear that this efficient energy transfer is being controlled by a strong cornering force or pull force which has overwhelmed the push force.

The previous plot showed both the magnitude and direction of the force and velocity vectors. Here a simplified diagram presents only the direction of the force and velocity vectors at a few points throughout the stroke. It is important to note that these forces are the total forces acting on the skate at "ground level" and are not necessarily the forces that the skater applies to the skate.



Conclusions

A completely efficient two-dimensional skating stroke was defined and analyzed. The stroke can be described as serpentine or sinusoidal-like. In this model the key to a complete transfer of sideways energy to forward depends on rotating the skate forward at the end of the stroke

creating a strong cornering force which pulls the skater forward and returns the skate to the center line. As there is no quantitative model for the cornering force the mapping between the force(s) the skater applies and the total force on the skate shown here cannot be adequately described. But it appears that the skater supplies a push and a force-couple or torque to rotate the skate and the wheel-ground interaction supplies the cornering force (qualitative model here). The advice "Push Hard, Turn Left" seems very appropriate here.

Unfortunately, skates do not turn well at very low speed so it seems unlikely that the necessary cornering force can be developed to make the starting stroke completely efficient. However, as the speed increases thereafter it is more likely that a very efficient stroke can be achieved. The double-push may well utilize such high-efficiency serpentine strokes. In Part 2 I intend to describe how the multistroke acceleration process leads to high speed.

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