The Fully Efficient Skating Stroke.
Part 2: Multistroke Acceleration.
c. P. J. Baum, January 2000.

12/19/1999

Introduction

In Part 1 of The Fully Efficient Stroke I looked at a single oscillating and accelerating stroke which began with zero velocity at time zero. It matched the performance of a linear accelerator operated at constant force and was appropriate for the start. However, as the velocity increases the power required increases also (Power=Force*Velocity) so this mode cannot be maintained for long. Once the cyclist reaches his power limit the operating mode must change to reflect the fact that he cannot perform at constant force any longer. This means that there is also a limit to the cornering force which the skater needs to develop to match the cyclist operating at the same power level. In Part 1 it was shown that even this stroke, operating at an ever-increasing power level, could be completely efficient so long as the appropriate cornering force could be developed.

In Part 2 I am looking at a series of strokes which can be maintained for an extended time. The emphasis here will be on the skater's power level and not the applied force as there is considerable data on the power levels skaters can maintain. For simplicity Part 2 will deal only with a constant power level.

Approach

In order to keep the two-dimensional skating stroke as efficient as the linear accelerator (e.g. bicycle), the sum of the sideways (Es) and forward energy (Ef) are matched to the energy (Elin) of the linear accelerator in the form:

$$Es + Ef = Elin;\ \text{or} \int \underline{Ps}*dt + Ef = Plin*t.$$

Here the sideways energy, Es, is based on a time integral over the average sideways stroke power, $\underline{Ps}$. Also used was Elin = Plin*t where Plin is the constant power level of the linear accelerator. We will also use the equation $\underline{Ps}$ = Plin so the power will be pumped in from the sideways stroke energy at the same rate that the linear accelerator operates. But to evaluate these equations we need a specific stroke model.

Stroke Model Details and Power Per Stroke

The stroke used here will be oscillating sinusoidally with a fixed (maximum) stroke width (Xsmax). Its sideways velocity pattern is defined by the cosine time dependence:

$$Vs(t)=Vsmax*\cos[Pi*t/(2T)].$$

Here Vsmax is the maximum sideways velocity of the stroke (3.70 m/s) here, Pi =3.14..., t=time (seconds), T=time of maximum stroke width (T=0.4 seconds in this model). In the figure below the sideways velocity is plotted in blue and the sideways position is plotted in red. The aqua region fills the velocity curve between adjacent forward-facings of the skate (T to 3T). At T and 3T the sideways velocity is zero, The stroke width is maximum, and the forward velocity (Vf) is maximum. In the middle of this range (2T) the sideways velocity is maximum at Vsmax while the skate is crossing the center line at Xs=0.

The aqua region is used to compute the average sideways energy per stroke (Es).

$$Es = \frac{\int_{T}^{3T} (M/2)*Vsmax^{2}*\cos^{2}[Pi*t/(2*T)]*dt}{\int_{T}^{3T} dt} = (M*Vsmax^{2})/4$$

Energy and Power Balance

From the previous integral we find the average sideways stroke power Ps = Es /T and set it equal to the power level of the matching linear accelerator:

$$Ps = Plin = (M*Vsmax^{2})/(4*T)$$

Next, the energy balance equation from the beginning of this section is solved for the forward energy, Ef:

$$Ef = Plin*t - Es$$

or,

$$Ef = (M*Vsmax^2)*t/(4*T) - (M/2)*(Vs(t))^2*\cos^2[Pi*t/(2*T)]$$

and the forward velocity then is given by:

$$Vf(t) = SQRT(Ef).$$

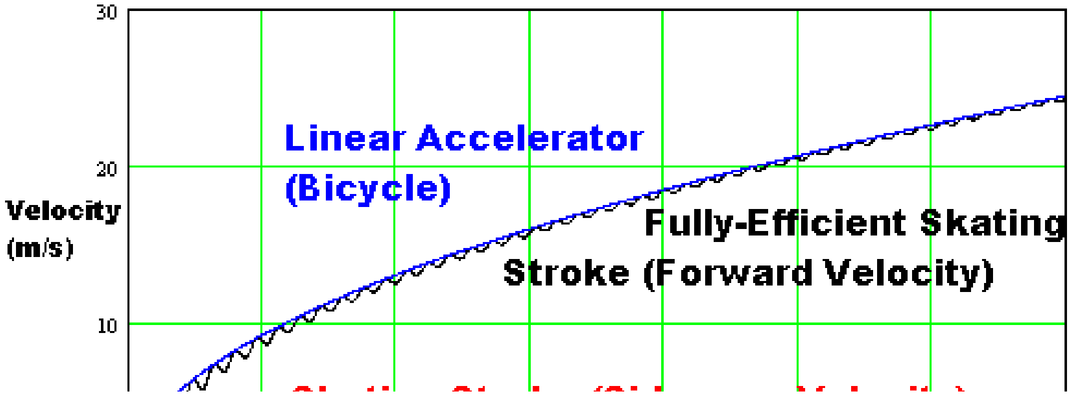
Next the results will be presented graphically.

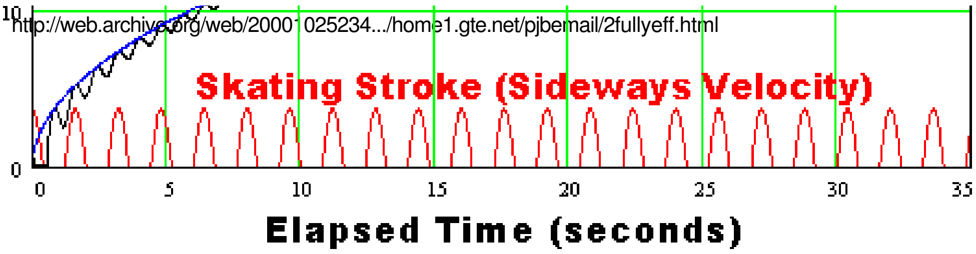
Results

The following results are based on the case: Power, Ps = 600 watts; maximum sideways velocity, Vsmax = 3.7 m/s; all drag forces = 0; skater's mass, M = 70 kg; time from center lime to maximum stroke width , T = 0.4 seconds. The chosen power is low for the start but high for an extended period of time where elite skaters might be more likely to generate 350-500 watts.

The energy equation above is plotted on the right. The blue line is the linear accelerator's energy as a function of time while the black curve shows the skater's forward energy. Because the energy is transferred from sideways (red) to forward at constant power the energy rises linearly on average. The situation is similar to that described by Mike Ryan (Accumulation of Speed, FaSST, Autumn 1999, Page 16) but the two-dimensional fully-efficient stroke accumulates energy only on average. Instantaneously the forward energy can decrease a little as the stroke turns sideways. The skater has some control over the peaks and valleys in the skating curve however as the energy is technically based on the position of the skate while the skater can maneuver his body to average out the bumps.

The square root of the energy yields the velocity which is plotted here vs. time. The blue curve is for the linear accelerator





while the black curve is the skater's forward velocity. The red curves are the sideways velocity (negative portion not shown). Velocity accumulates on average. The velocity keeps increasing without limit because no wind drag or rolling resistance has been included in this calculation.

Here the forward velocity has been integrated over time to give the distance travelled as a function of time. The time to travel 100 meters is a little under 8 seconds at this power level which is barely below Chad Hedrick's time. The time to travel 500 meters is a little under 25 seconds. Shimizu can skate 500 meters in 35 seconds. So once wind drag is factored in it is likely that he is skating with a fairly high efficiency even using a classic stroke. (The classic stroke seems to suffer more from a low duty cycle than a low efficiency).

Discussion And Conclusions

In Part 2 of the Fully-Efficient Stroke it was shown how (at constant power) energy and speed build up from the repeated transfer of sideways power into the forward direction. In Part 1 it was shown how the development of a strong cornering force enabled the single stroke to be fully-efficient. Possibly more controversial are some of the observations made during the course of this analysis. In the subsection on the Stroke Model above on this page the sideways velocity was defined to have a cosine form. So the sideways velocity is zero at the stroke width extremes (T, 3T...) and it has a maximum along the skater's center line (2T, 4T...). This is quite different from the classic stroke where the sideways velocity is zero at the skaters center line. And if you differentiate the sideways velocity with respect to time to obtain the sideways acceleration for the fully-efficient model you find that the skate accelerates toward the center line (at *all* times). That means that the skate is experiencing a continuous pull force and never a push force. At first this seems incorrect as in Part 1 the skate experienced a push force on the start. But -- the start had a zero sideways velocity at the center line like a classic stroke and it had a zero sideways velocity at the maximum stroke width. So logically it had to accelerate and then deccelerate to accomplish anything -- hence the push and then the pull. But if you remove the requirement of zero sideways velocity at the center line (setting the skate down straight forward) the need for an overall

push force seems to vanish.

To see if this model had anything in common with real skating I examined some video taken of Chad Hedrick. The image to the right was prepared from a video of Chad kindly supplied by Kim Hendrikse. The video was taken with a zoom lens from directly in front of Chad to record his sideways motion and here I cropped it down to show only his boots, frames, and wheels. The video comprising almost one full time cycle of one version of his double-push was exported to a vertical filmstrip and compressed vertically so you can't pick out much detail in this form. Here the serpentine curves are the time-tracks of Chad's feet with time increasing downwards. The apparent deviation from sinusoidal at the far left occurs when Chad lifts his right skate off the ground and that skate stops developing cornering force. The inside tracks from the two feet merge together to form a track close to sinusoidal and looking a lot like the fully efficient stroke. From these tracks it is clear that the sideways velocity goes to zero only at the left and right extreme points. Clearly the skates continue to move across the center line (not drawn) and from my examination of data read from a larger version of this image I was able to construct a sideways velocity pattern in reasonable agreement with the cosine form I used for velocity. The velocity is the first time derivative of the skate positions read from the image. The force/acceleration comes from the second derivative of the position so the slightest noise in the position readings produce spikes which might be pushes. However, from the fact that the sideways velocity is zero at the maximum stroke width and is maximum at the center line means that the trend of the force must be toward the center line (a pull force). To determine whether there are ever any real pushes in Chad's skating method will require a study of more strokes with data averaging to remove most of the noise. But, within measurement error I find conclusive evidence for a quite extended pull force and possible transient pushes although the pushes appear to be near the noise level. So far the model here seems to meet the reality test.

Click to go back to the *INDEX*
