

The Fully Efficient Skating Stroke. Part 3: Classic vs. Double Push.

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Introduction

Recently I looked at a Fully Efficient Stroke finding two-dimensional stroke motions corresponding to a form of the double push. That solution had only one mass as it was assumed that the body and powering skate moved completely in phase. Here it will be necessary to introduce two masses so that the motions of the skater's body and powering skate and leg can be out of phase. The solutions here will be at constant force (increasing power) and correspond to several types of fully efficient stroke including Classic. From the energy-conserving two-dimensional model, the concepts of "Single Push" and "Double Push" are re-examined leading to a new definition of "Double Push" as well as to the identification of a "Triple Push". In Single Push the powering is done by one leg only (at a time). In Double Push the powering is done by one leg and the body simultaneously. Triple push is powering by two legs and the body simultaneously. The consequences for top speed are briefly discussed.

Approach

In the previous analysis I matched the skater's energy to a linear accelerator. If the linear accelerator's force is F and the mass is M then the linear acceleration is $a=F/m$, its velocity is $v=a*t$ and its energy is $E_{lin}=(Mv^2)/2$. Since there is only motion forward and I am not considering drag forces this linear acceleration will be fully or 100% efficient. The method used is to look for two-dimensional skating strokes which will always have the same energy as the linearly accelerated object. Now the skater will produce sideways energy (E_s) as well as forward energy (E_f) through his forward velocity (V_f) and sideways velocity (V_s). So to match the linear accelerator I require that the stroke satisfy

$$E_f + E_s = E_{lin}, \text{ or, } V_f^2 + V_s^2 = v^2 = (a*t)^2 = (Force/M)^2*t^2.$$

Analysis And Analytical Results

First I very briefly present the previous solution applicable to a form of Double-Push and then move on to Classic Stroke-like solutions.

1. Double-Push Stroke

The previous Double-Push-like solution to the energy equation in the previous section is

$$V_f=(F*t/M)*\sin[\text{Pi}*t/(2*T)] \text{ along with } V_s=(F*t/M)*\cos[\text{Pi}*t/(2*T)].$$

or of the form: $V_f=v\sin(t)$, and, $V_s=v*\cos(t)$, where $v\sim t$ for constant force. Here $\text{Pi}=3.14\dots$, t =time, T =time of maximum stroke width. M is the total mass of the skater plus skates.*

2. Classic Stroke

For The Classic Stroke we need to introduce another mass. The mass of

the skater plus skates will be "M" as before but I introduce "m" which is the mass of the powering leg plus skate. The two components then will be m, which refers to the skate (plus leg) and M-m, which refers to everything else (skater + two skates - powering leg - powering skate). As in the double-push case I assume that all masses move forward at the same speed: Vf (Vf=vf). The skater can adjust his body position to stay exactly even with his powering skate. In terms of sideways motion the powering leg and skate (m) will move sideways at the speed vs while the body (M-m) will move at the speed Vs. The fact that the powering skate moves exactly sideways leads some to call this a linear stroke. However, I consider it a nonlinear stroke because it will be found to be straight-line only in the frame accelerating with the skater. In the frame of the ground it is a curve (nonlinear). Hence the skate is turning its direction even as it pushes straight sideways in the accelerating frame.

I consider two classic cases: first, the sideways momentum is conserved in the form $m*vs + (M-m)*Vs = 0$. Which is the relation which would hold if you stroked to the side while neither skate rested on the ground; i.e. no momentum transfer to the ground as the body and leg/skate have equal and opposite momenta. The second case will assume $vs=0$, i.e. no sideways motion of the body at all. To do this the skate under the body has to react with the ground so as to cancel the momentum the powering skate would otherwise deliver.

2a. Skater Sideways Velocity from Momentum Conservation

$$M*vf^2 + (M-m)*Vs^2 + m*vs^2 = M*v^2 .$$

$$(M-m)*Vs + m*vs = 0$$

$$M*vf^2 + [M*m/(M-m)]*vs^2 = M*v^2 .$$

The solution is: $vf=v\sin(t)$, with, $vs=[(M-m)/m]^{1/2} *v*\cos(t)$,
along with: $Vf=v*\sin(t)$, and, $Vs=-[m/(M-m)]^{1/2} *v*\cos(t)$.*

2b. Skater Sideways Velocity Is Zero

$$M*vf^2 + m*vs^2 = M*v^2 .$$

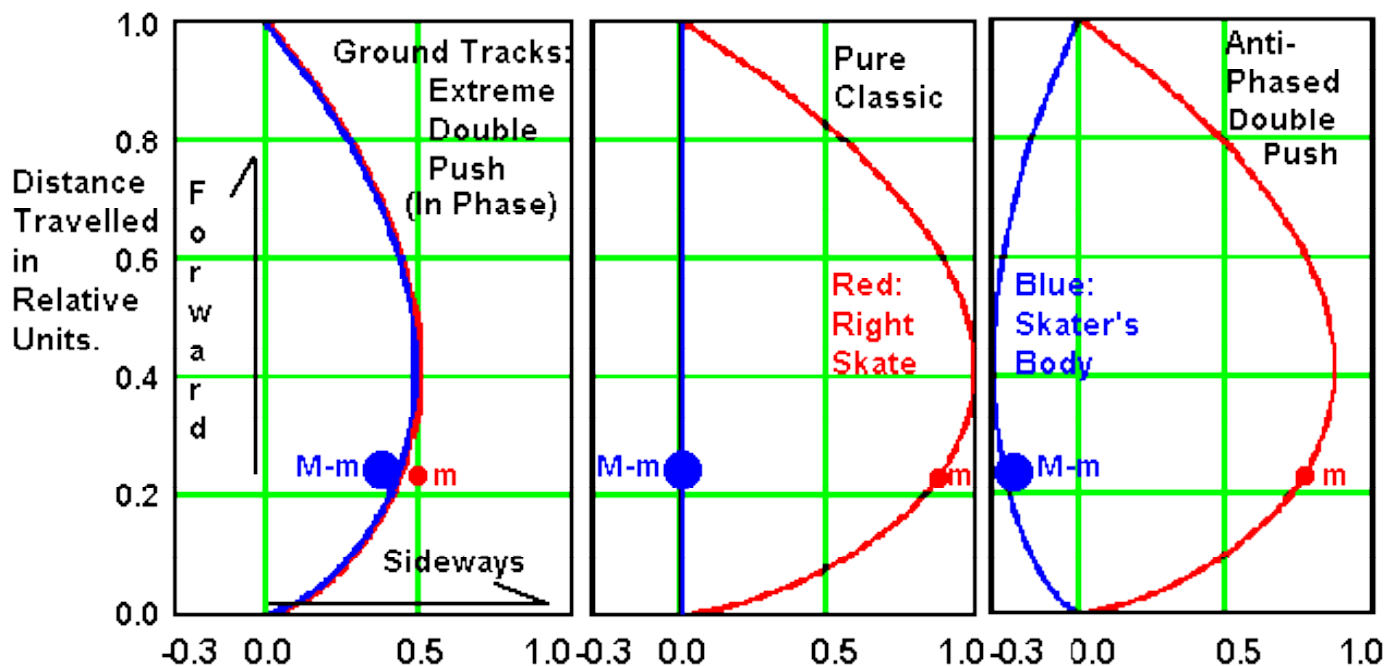
The solution is: $vf=v\sin(t)$, with, $vs=(M/m)^{1/2} *v*\cos(t)$,
along with: $Vf=v*\sin(t)$, and, $Vs=0$.*

The results are plotted in the next section.

Graphical Results

The mathematical solutions found in the previous section are plotted below. They are all presented on the same (dimensionless) scale with the forward space axis pointing up and the sideways space axis pointing to the right. All the strokes are done at the same Force, the same total mass, and the same stroke frequency. The only parameter which varies with forward motion is the stroke width or sideways position. Each example case uses the particular mass ratio $M/m = 4$. The case on the left looks at the Double Push limit where the body and skate have equal sideways excursion and they are in phase. The middle panel is pure Classic where there is no sideways motion of the body. The right panel I am calling an antiphased Double Push now instead of the momentum balanced Classic stroke it was described as above. The reasons for this

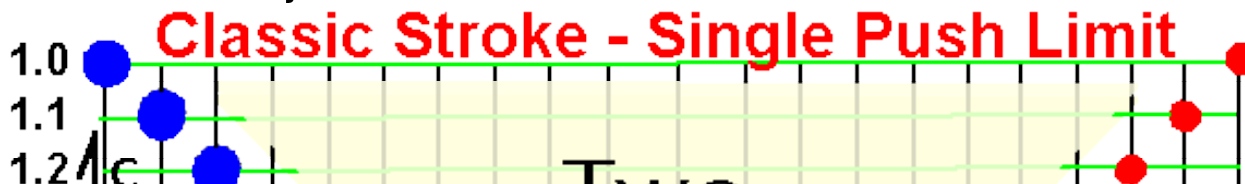
will become apparent shortly.

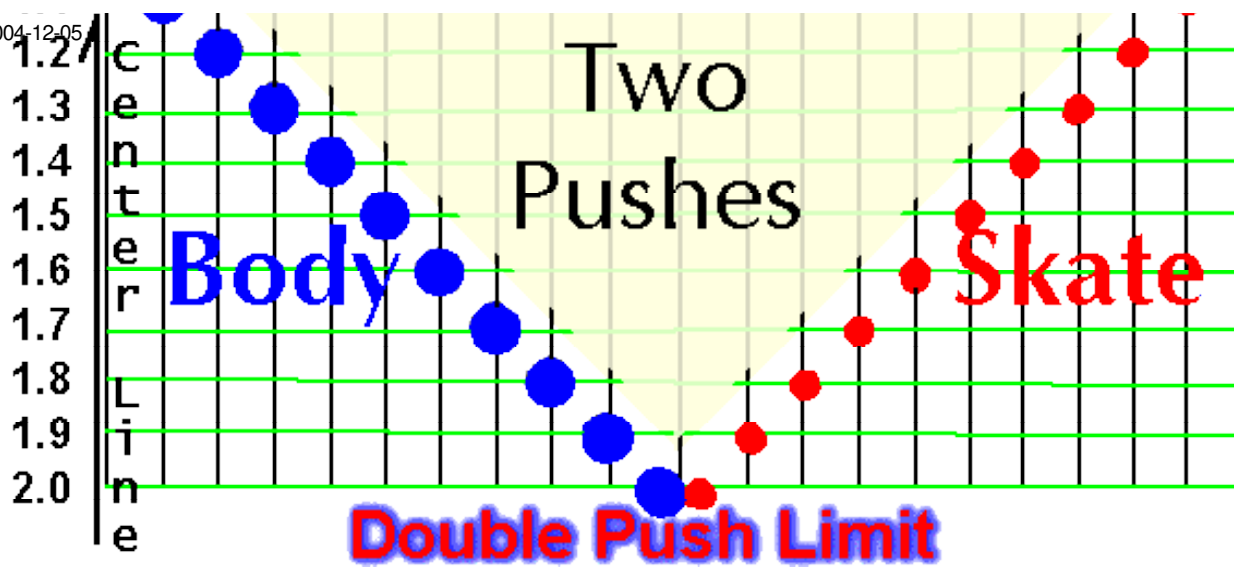


Note that for this case ($M/m=4$) that the stroke width for the Double Push limit is about 0.5 whereas for the Pure Classic it is about 1.0. So it is apparent that for the same acceleration the legs only have to push half as far in this Double Push as they do in the Pure Classic limit. The reason for this is that the Double Push indeed has two pushes and in this limit they are two equal pushes -- one by the body and one by the leg. The body's sideways momentum adds to that of the leg as it pushes. So the leg delivers full force to the ground but only generates half of it in the leg muscles.

The panel on the right shows the case calculated from momentum balance between the body and the powering leg. The leg push here has moved the body out of phase with the leg and across the skater's center line. I think of this as an antiphased Double Push or a negative Double Push because the body's momentum now subtracts from the leg push as evidenced by the fact that the distance between the right skate and the body is now even larger than in the Pure Classic case. Because of these facts the right panel does not seem practically useful except as a concept to be considered.

The Double Push limit (Left, above) and the Pure Classic stroke (Center, above) can be thought of as the limits of speedskating. The figure below shows part of the continuum between these two styles. Pure Classic is on the top line, labelled 1.0 for single push, with the body shown on the far left and the skate on the far right. The skater's center line for his forward motion is upwards along the arrow. The stroke has pushed out to the right. Each horizontal line shows only the maximum stroke at one instant of time. At the very bottom of the graph we find two equal pushes by the body and the powering leg and it is labelled 2.0 for the Double Push limit. It is clear that the body's and leg's stroke can take on a range of values and they are shown on the lines which are labelled 1.1-1.9. For example, in the 1.1 case there are two pushes (so I place it in the inverted triangle region for Double Push). But the 1.1 stroke is very near the single push limit, 1.0, because the body's push is rather weak with a short sideways excursion.





Discussion and Conclusions

Because of the range of ways in which the two pushes can be combined it is clear that there can be more than one style of Double Push. The classification scheme presented in the previous section allows us to categorize the skating style between 1.0 and 2.0 based on the amount of sideways push delivered by the skater's body. There is no body push at 1.0 while the body push equals the leg push at 2.0. The advantage of the Double Push is that it spares the leg muscles as they do not have to work as hard as they would in classic. But there is a downside also: Because the powering skate is closer to the body in Double Push the knees need to be bent less and the skater tends to stand up more and more as the stroke increases from 1.0 to 2.0. Consequently the skater's cross section for wind drag increases in Double Push. And, in addition, the body's path becomes more and more serpentine as the Double Push limit is approached so that the path length travelled through the air becomes considerably larger than for the single push stroke. Then it becomes apparent that the highest strokes (approaching 2.0) will only be useful at fairly low speeds and as the speed increases the skater must move his stroke back closer to 1.0 or Classic in order to avoid serious wind drag problems.

Additionally, It is noted that there is still another power source which has not been included here. That is the second leg which can also power in phase with the body and the first leg. I would call this a "Triple Push". When using both legs to power there isn't much rest and burnout is likely to occur but the Triple Push may be useful for short bursts of acceleration.

Finally, I note that while conceptually very helpful, this model is only two-dimensional so leaning is only approximated by moving the body's mass toward the center line. Gravity, with its pendulum-like effects, has not been considered at all yet. It will be interesting to see how these concepts are altered when someone gets fully-efficient three-dimensional solutions. Furthermore, only constant force was considered here and it is quite possible that the way skater's actually power might lead to nonconstant acceleration during a stroke.