

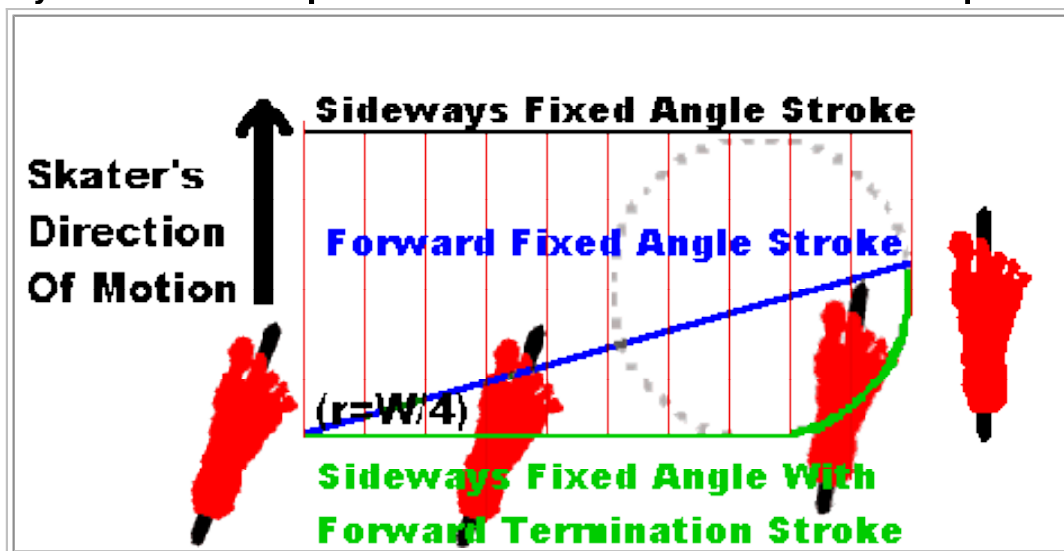
# "Exact" Nonlinear Stroke Model

c. P.J. Baum, December 1999.

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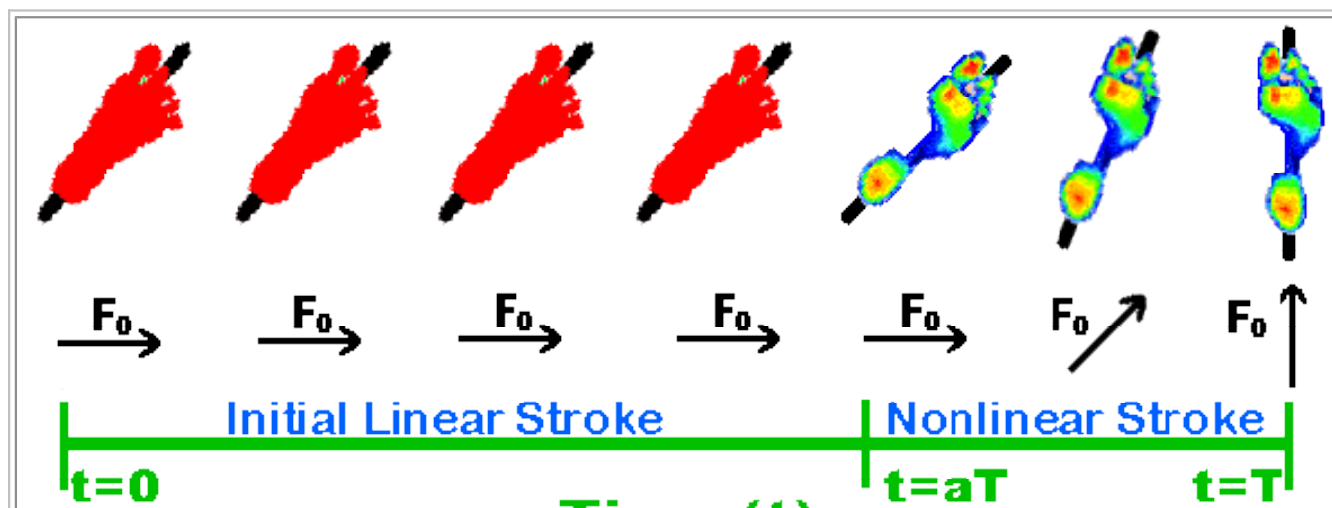
## 1. Introduction

On an earlier page I had discussed coupling a linear sideways stroke with a forward turning stroke to increase the energy output of a skating stroke. Such a situation is shown below where the sideways skate angle and stroke position are shown as the right foot moves to the right. The analysis was an estimate only and the intent of this page is to quantitatively describe the stroke. The word "Exact" in the title is in quotes because when all is said the solution turns out to be close to the one we wanted but not quite the same. Nevertheless, the solution is highly instructive and points to the correct answer for several questions.



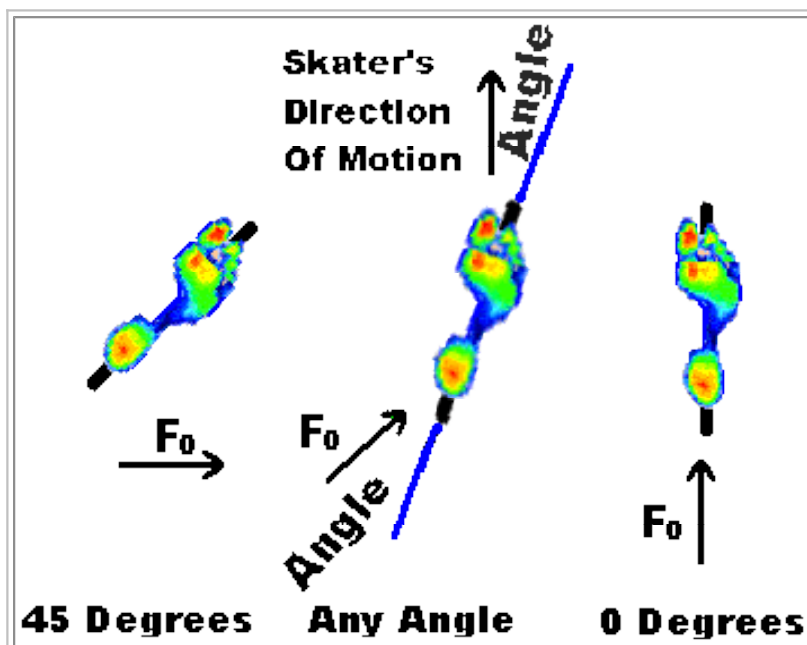
## 2. The Stroke Model

In the figure below the proposed stroke is shown with time now plotted across the bottom and not distance as before. The stroke is completed in time T and begins with a linear stroke (red feet) from time  $t=0$  up to  $t=aT$  where a is some number between zero and one to be determined. After that time the stroke is completed with a nonlinear turning stroke (multicolored feet).



# Time (t)

The direction of the force ( $F_0$ ) the skater applies is also shown. The force model comes from a forward-stroking model I used earlier where the problem is reduced to only one angle because the angle between the force and the skate is equal to the angle between the skate and the direction of motion (forward)-- see below.



The stroke starts out sideways and turns to straight ahead (Angle starts at 45 degrees and turns to 0 degrees). So the turning stroke is modelled as a sequence of linear strokes of different angle. No cornering properties of the skate or wheels is included but the force splitting is based on an absence of linear sliding. It turns out, that the linear stroke always loses (in this model anyway) and that the skate ends the stroke at highest velocity for  $\alpha=0$  (no linear portion at all). So the solution now becomes simpler with only a nonlinear portion. The skate angle as a function of time,  $t$ , is:

$$\text{Angle}(t) = [\text{Pi}/4] * [1 - t/T]$$

where  $\text{Pi}/4$  is 45 degrees expressed in radians. Now the model has the forward force varying as  $\cos(\text{Angle})^2$  while the sideways force varies as  $\sin(\text{Angle})^2$ . So the terminal forward speed of the skate will be given by a time integral of  $\cos(\text{Angle})^2$ .

## 2a. Results: Stroke Efficiency With A "Simple Pull"

The exact solution to the integral equation for forward velocity is:

$$v_f(t) = [F_0/2M] * [t - \{2T/\text{Pi}\} * \{\cos(\text{Pi}/2 * t/T)\} + 2T/\text{Pi}]$$

which has as its value at  $t=T$

$$v_f(T) = [F_0 * T/M] * [1/2 + 1/\text{Pi}] = 0.818 * [F_0 * T/M]$$

So this model ended up with **81.8% of the maximum velocity** available or **66.9% of the maximum energy** available ( $v_f^2$ ).

**This is a little disappointing but the reason for only a modest improvement over a linear stroke becomes clearer when we find that at time  $t=T$  that the sideways energy has not dropped to zero.**

**In fact, the force must be applied for an additional 8 degrees (turning back toward the skater's center line) before the sideways energy drops to zero. Then the forward *velocity increases to 91.9%* of the maximum available and the *energy increases to 84.5%* (even when the additional energy input from the longer stroke time is considered). If you look at the force arrows in the diagrams above only outward push forces were applied and it was not until an inward "simple pull" force was used (for the additional 8 degrees) that the sideways motion was really halted. We had to oversteer the skate to get it to fully turn the corner. So this model is for a skate/skater with very poor cornering characteristics.**

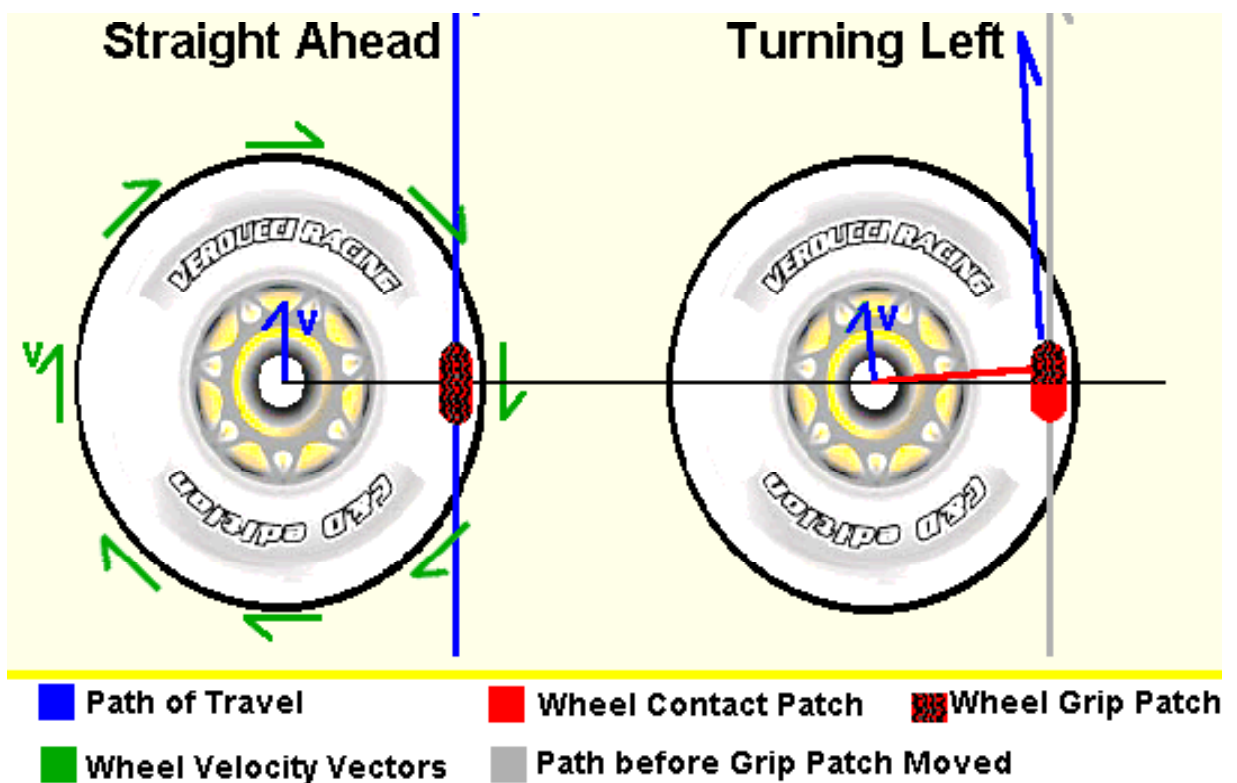
## **2b. Efficiency Using A "Complex Pull"**

### **2b1. How the "Complex Pull" Results from the "Cornering Force"**

**The model above is rather like a poor-handling car where the cornering can be improved by taking advantage of the tire's "Grip Angle" or "cornering force" [The Physics of Racing, Part 10: Grip Angle by Brian Beckman]. As Beckman explains, turning the steering wheel does not necessarily turn the car unless a real force (cornering force) is developed to pull the tire into the turn. The difference between the direction you steered and the direction the car moves is the "grip angle" and it will be near zero for a car which handles well. This cornering force is due to a differential grip across the tire's contact patch caused by the torsional stress on the tire and is independent of any linear (sideways) sliding or "slip". Unfortunately I have found no quantitative theory or model for the cornering force so I can only describe it qualitatively at this time. The figure below shows the turning concept I developed earlier for the inline skate wheel. The normal "push" force has caused the skate to move straight ahead along the blue line in the left panel and here the grip (tread pattern) is uniform throughout the contact patch. As the skater executes the turn in the right panel the torsion increases in the contact patch causing the grip to move forward. This results in the wheel being "pulled" left away from the direction a linear "push" would move it.**

**Inline Skate:  
Straight Ahead** 

**Turning Left** 



pjb- Speedskating Santa Barbara 2-21-99 Inline Turning Concept  
**Inline skate cornering has not progressed beyond the qualitative model so no quantitative results for efficiency can be obtained now. It may be noted however, that the pull from the cornering force develops a velocity component perpendicular to the skate which is absent in the linear push. This force first pulls the skate forward, assisting the push in propulsion, and then pulls the skate back to the skater's center line shortening the stroke time.**

**On ice or snow the cornering force is developed through the "carving turn" whereas on inlines the skater manipulates the wheel's contact patches to develop the cornering force. The skater pushes down with the heel to enlarge the rear contact patch and increase its torsion while rolling. The larger contact patch means the turn can be sharper if enough torsion is developed. In addition the outward heel push increases the rear wheel's contact patch torsion still more and the result is the cornering force which pulls you around the corner while the push helps the skate ahead.**

### 3. Summary And Conclusions

- **The analysis began with a linear stroke coupled to a nonlinear stroke. Soon only the nonlinear stroke was retained because *the linear stroke always provided less forward propulsion* within the parameters of this model.**
  - **Consequently it looks like *the stroke should always be turning* (forward).**
- **The nonlinear stroke was modelled as a sequence of (turning) linear strokes of various angle. This gave a velocity efficiency of 82% and an energy efficiency of 67%.**
- **When a simple pull force was added to the stroke to halt the**

**sideways motion the velocity efficiency increased to 92% and the energy efficiency to 85%.**

- **The desired complex pull force (which experienced skaters seem to use) was identified as the cornering force which relies on torsional effects in the contact patch of the skate wheels.**
- **A qualitative model for the cornering is available but not a quantitative one.**
- **Since the linear (sideways) push force produces sideways and forward motion of the skate it can only use 50% of the skater's power to produce forward propulsion. To increase the forward power it looks desirable to use the cornering pull force to provide more propulsion (possibly nearly the other 50% of the skater's power).**
- **The reason it has taken so long to identify the pull force is because the push, the turn, and the pull all occur simultaneously in the nonlinear stroke. The skater provides the push and the skater-skate-ground interaction converts part of the push into the pull.**
- **To nail down the maximum efficiency quantitatively will require an accurate model for the skate's cornering force.**

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