The Second Push In The Double Push.

c. P. J. Baum, October 2000.

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Introduction

While speedskaters seem to agree on the meaning of "single push" the double push has been a continuing puzzle (at least to me). This page attempts to clarify the meaning of the second push in the double push. There will be little math here as I am laying out the beginning of the solution in qualitative terms and leave the details to another day. But I will refer to several types of pendulums for analogy to skating. Anyone who has skated the double-push will recognize the importance of gravity (the "fall"), and the oscillatory sideways motion which lead to a pendulum analogy. I present a very brief review of the pendulum and then move on to its skating application. Three types of body push are finally identified as involved in the second-push (the leg push being the first-push).

I am using the word "push" here in the sense of a force. This does not mean that "pulls" are unimportant (except possibly in ice speedskating where it is not clear that the skate can turn or grip for a pull). My earlier study of skating efficiency shows that pushes and pulls are of equal importance at 100% efficiency and 100% duty cycle. Some inline skaters use an explicit muscle pull while others use mainly a muscle-push and rely on the cornering properties of the inline wheel to provide the pull.

Background: Pendulum Review

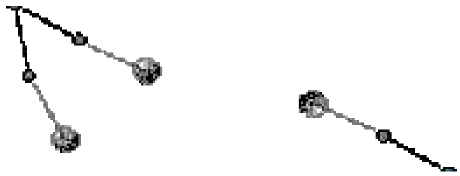
Pendulum Types

Several types of pendulum are illustrated below. Everyone has probably seen the simple pendulum on the left where a mass (red ball) is suspended from a string and gravity makes it oscillate if it is released from a high position. But the skater cannot be suspended from above so the next animation shows the pendulum turned on its head for the simple inverted pendulum. The inverted pendulum requires that the string be replaced with a thin stiff rod and that appropriate sideways forces be applied at ground level or the inverted pendulum will fall to the ground without executing even one oscillation. We can think of the inverted pendulum acted on by the ground reaction force to the skater's (single-) push force.

The third and fourth panels above illustrate the torsional or rotational pendulum and the inverted torsional pendulum. Here the globe oscillates because the suspension has a spring constant which restores the globe if it is displaced rotationally.

Finally, consider the ordinary double-pendulum (below, left) and it's

inverse, the double inverted pendulum (below, right). This illustrates how two pushes can add to produce a single ground reaction force which is powered by two separate entities (the upper and the lower pendula). Identifying the upper push force will be the goal of this page.



The above solution was obtained using the "Albert" program. You can modify the parameters of the ordinary double pendulum by downloading "ALBERT" here.

What Drives a Pendulum?

Since gravity is the only force acting on a simple pendulum it must be the driving force. However we need to know just what it drives. The simple pendulum has an oscillating motion with frequency of oscillation depending on the square root of g/L (g is gravitational acceleration, L is the length of the suspension string, cable, or rod). So gravity drives a steady state oscillation back and forth -- except that a real pendulum has damping from air and suspension friction. So a real pendulum has a damped oscillation and gravity does not only not drive an accelerating motion, it cannot even maintain a steady motion unless some external driving force is added, say from a pulsed electromagnet to replace the friction losses. (Gravity does accelerate the simple pendulum downwards but it deccelerates the pendulum when it moves upwards so that the average state has no acceleration unless the pendulum moves downhill, as on wheels.) So gravity will not be the driver I am interested in. The external driving force which adds back what energy friction "eats up" will be closer to the driver of interest.

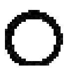
Now when we come to the inverted pendulum there is an additional driving force: namely, there must be forces at ground level to stabilize the pendulum rod or the pendulum will immediately fall to the ground. Later on this page the leg force (Speedskaters's Single Push) and the ground reaction force to this push will serve this role. So the skater's leg push will be the primary driver of interest in that case. Gravity will affect the skating dynamics and provide the required wheel grip but is not the primary driver here.

In the case of a torsional or rotary pendulum gravity is of no consequence since there is no height change. The double pendulum will be related to the double push in the next section.

Single Pushes and Double Pushes

Speedskater's Single Push

In the Speedskater's single push or classic style of skating the leg is the primary driver. In this style the pendulum analogy is obscured because the suspension (rod length = leg extension) changes in length in a manner so that the skater's body stays at nearly a constant height throughout the stroke.

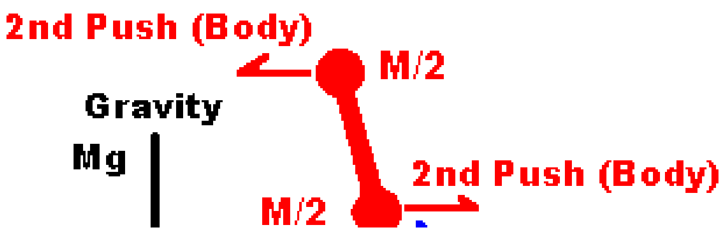
Beachskater's Single Push

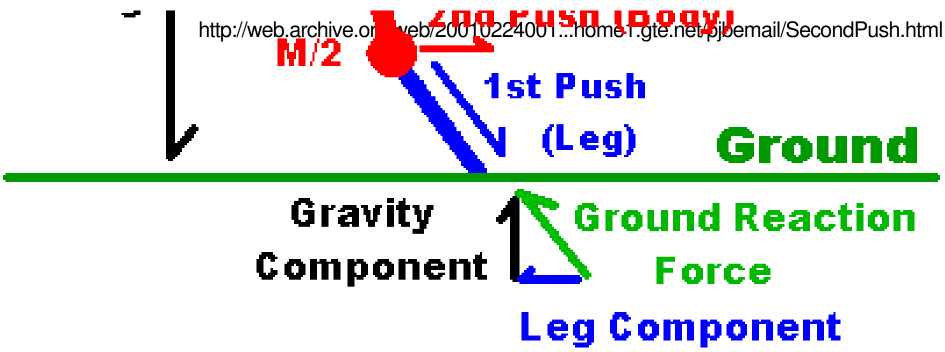
The beginning skater powers in a completely different manner than the classic speedskater. First they try to walk on their skates. However, beach skaters soon give this up and many of them begin to do a (rather modest) type of pendulum stroke. They do not bend down at all and they do not push along their leg length. They learn that by transferring their weight from side to side, one skate to the other, that they can propel themselves at modest speed with very little effort. They are afraid to lean very much but they do oscillate like an inverted pendulum with a shallow oscillation angle. So where does the power come from in this case? Well, I rule out gravity. Stand up straight over your two feet and observe how long it takes to lean far enough to the side to feel like you're falling. While sober you may feel like the leaning tower of Pisa -- it takes quite a while to fall as gravity is not very effective in producing leaning while you are standing nearly straight up. In fact I find that I need to actively lean using my upper body muscles and then when I am around 15 degrees from vertical I begin to feel a sensation like gravity might make me fall. So I conclude that the Beachskater's Single Push is driven by upper body muscles and not by gravity or the leg muscles.

Double Push: Model For The Second Push

In this section I will argue that the two types of single push shown above can be combined synergistically to form a double push. But first let's see Chad do a double push. This is a degraded clip from Kim Hendrikse's video "chadstreet.mpg", and I have superimposed blue lines to accentuate the body motion.

The blue line through his upper body looks for all the world like a gravity-driven pendulum as Chad's double push motion is so smooth. But below I will sketch what I think really happens here. I have divided the upper body mass (M) into two equal parts, one of which is located at the shoulders and the other at the hips. (The upper body is color-coded red and the leg is blue below). Gravity pushes down and the leg has a component which pushes to the side. The ground reaction forces are shown for gravity and for the leg push. However, consider if Chad uses his upper body muscles to force a lean (hips right, shoulders left). "Lean" is used here in a muscular-active sense here and not in the gravity-active sense of the Leaning Tower of Pisa. The result would be that an additional force would be transmitted to the ground through the leg and an additional ground reaction force would be developed from the upper body motion (not drawn below).

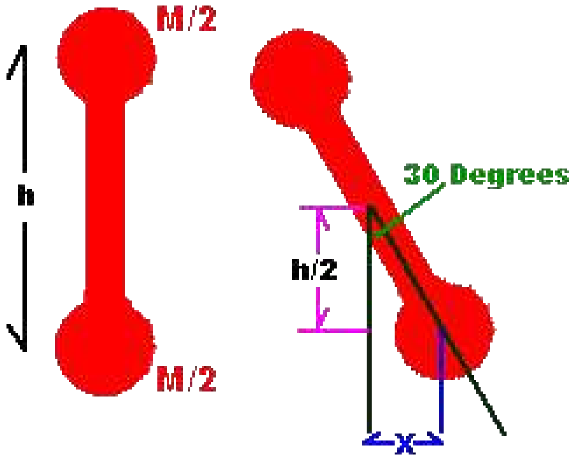




Now this would be one way of supplying a second force to energize a driven double-inverted-pendulum model of skating. But there are also a couple other ways to supply the second push: The hips can rotate to supply a torsional motion for the upper pendulum or the skater can move his upper body up and down instead of left-right. As I look at the video clips of various skaters I find all three methods of driving the second push in use. Chad Hedrick clearly uses the left-right body-lean and also some torsional body-driving. Barry Publow can be seen using the left-right body-lean as well as the up-down body-driving in this animation.

Estimated Magnitude of the Second Push

The skater's upper body model is shown on the right. Here the upper M/2 represents half of the torso mass located around the shoulders and the lower M/2 is the other half located at the hips. The distance from the shoulders to the hips is labelled as "h" and the sideways lean excursion is "x" (relative to the center of the torso). From the video clip Chad's lean angle reaches about 30 degrees and I use h=50cm to calculate "x": x = h*tan(30 deg). So x = 25*.577 =14.4 cm or x=0.144 meters.



The power, P, associated with the torso lean is P = Energy/Time = $(M/2)v^2/T$ where T will be the time to move from the center line to maximum stroke excursion (the quarter-cycle time of the stroke). Using v=x/T the power becomes

$$P = (M/2)x^2/T^3$$

So choosing M=120 pounds = 260 kg, M/2=130kg, x=0.144m, T=0.31 Seconds, we find P= 90 Watts. Here T came from an analysis of Chad's video ground tracks. The full cycle time of his stroke was 1.26 seconds if the video camera had a standard frame rate of 30 frames per second. So the quarter cycle time was 0.31 seconds.

Summary And Conclusions

A qualitative model for driving the double-push was outlined, relying on an analogy with a double-inverted-pendulum. The second-push comes from motion of the body which powers through the leg which provided the first-push. While the first push is provided by the leg push, the second-push can come from three different forms of body motion:

1. **Sideways torso lean**
2. **Torsional or rotational body motion**
3. **Up-Down torso lean**

Chad Hedrick and Barry Publow use different combinations of these body motions. It is not clear yet just what body motion or combination of motions is optimum. And since there are no measurements separating the three second-push modes I can only surmise that the most powerful (highest acceleration) mode might involve the sideways lean. But here the skater is standing nearly vertical so that the wind drag may also be maximum and this would limit the top speed of this form of second-push. The torsional mode can be done while the skater is bending down in a reasonably low position and may have the lowest wind drag which would make it the highest speed form of second push.

The power estimate above showed that apparently small body motion (14 cm or less than 6 inches) can provide significant power (90 watts here) because the body has so much mass. Consequently the second-push or "body stroke" does not need to be as wide as the first-push or leg stroke.

The gravity-powered pendulum loses energy to drag. The skater's inertially-mass driven inverted-pendulum can actually gain energy because it is powered by muscles. Gravity aids in setting up the the body's oscillation frequency which gets synchronized with the stroke frequency. Once the sideways oscillation energy is large enough the Skater turns the excess sideways energy forward maintaining a constant stroke width and accelerating forward (or maintaining a constant forward speed against wind drag). The downside of the second-push is that it increases the wind drag at the same time it increases the propulsion power. Not much can be said quantitatively about this at present, however.

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